

BT-4/J07

8758

Mathematics-III

Paper-Math-201 E

Time : Three Hours]

[Maximum Marks : 100

Note :- Attempt any FIVE questions, selecting at least ONE from each section.

## SECTION-A

1. (a) Expand  $f(x) = x \sin x$ ;  $0 < x < 2\pi$  in a Fourier series. 10(b) If  $f(x) = x$ ;  $0 < x < \frac{\pi}{2}$ 

$$= \pi - x \quad \frac{\pi}{2} < x < \pi$$

Show that

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi} \left[ \frac{1}{1^2} \cos 2x + \frac{1}{3^2} \cos 6x + \frac{1}{5^2} \cos 10x + \dots \right]$$

2. (a) Find the Fourier sine and cosine transform of  $f(x) = e^{-x}$  ( $x > 0$ ).

(b) If the initial temperature of an infinite bar is given by

$$\theta(x) = \begin{cases} \theta_0 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

determine the temperature at any point  $x$  and at any instant.

## SECTION-B

(a) If  $\sin^{-1}(x + iy) = \log(A + iB)$ , show that

$$\frac{x^2}{\sin^2 u} + \frac{y^2}{\cos^2 u} = 1, \text{ where } A^2 + B^2 = e^u$$



3. Show that the function  $f(z) = \sqrt{xy}$  is not analytic at the origin even though C-R equations satisfies thereat. 10
4. (a) Determine the Regular function whose imaginary part is  $e^{-x} (x \sin y - y \cos y)$ . 10
- (b) Show that the transformation

$W = 2 + \frac{a^2}{z}$  transforms circle with origin at the centre in the  $z$  plane into co-axial concentric, confocal ellipses in the  $w$ -plane 10

### SECTION-C

5. (a) The contents of three urns are : 1 white, 2 red, 3 green balls ; second 2 white, 1 red, 1 green ball and third have 4 white, 5 red, 3 green balls. Two balls are drawn from an urn chosen at random. These are found to be one white and one green. Find the probability that the balls so drawn came from the third urn. 10
- (b) The probability density  $p(x)$  of a continuous random variable is given by

$$p(x) = y_0 e^{-|x|}, \quad -\infty < x < \infty.$$

Prove that  $Y_0 = \frac{1}{2}$ . Find the mean and variance of the distribution. 10

6. (a) Fit a Poisson distribution to the following data given the no. of cells in the area covered by 100 sq. mm.

No. of Cells per sq.	0	1	2	3	4	5	6	7	8	9	10
No. of Square	103	143	98	42	8	4	2	0	0	0	0

- (b) Show that the standard deviation for a Normal Distribution is approximately 25% more than the mean deviation. 10



7. (a) Graphically solve the LPP

$$\text{Minimize } Z = 6x + 14y$$

$$\text{Subject to } 5x + 4y \geq 60$$

$$3x + 7y \leq 84$$

$$x + 2y \geq 18$$

$$x, y \geq 0.$$

(b) Solve the LPP using Simplex method

$$\text{Max. } Z = 10x_1 + x_2 + 2x_3$$

$$\text{Subject to } x_1 + x_2 + 2x_3 \leq 10$$

$$4x_1 + x_2 + x_3 \leq 20$$

$$x_1, x_2, x_3 \geq 0$$

8. (a) Using Dual Simplex method

$$\text{Minimize } Z = 2x_1 + x_2$$

$$\text{Subject to } 5x_1 + x_2 \geq 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \geq 3$$

$$x_1, x_2 \geq 0.$$

(b) Illustrate the following and their importance in dealing L.P.P.

(i) Slack / Surplus Variables

(ii) Artificial Variable

(iii) Basic Variable

(iv) Non-Basic Variable

(v) Degeneracy in L.P.P.